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# Modelling variably saturated flow using cellular automata

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## Abstract

The use of Cellular Automata is extended in various disciplines for the modeling of complex system procedures. Their inherent simplicity and their natural parallelism make them a very efficient tool for the simulation of large scale physical phenomena. We explore the framework of Cellular Automata to develop a physically based model for the spatial and temporal prediction of shallow landslides. Particular weight is given to the modeling of hydrological processes in order to investigate the hydrological triggering mechanisms and the importance of continuous modeling of water balance to detect timing and location of soil slips occurrences. Specifically, the 3D flow of water and the resulting water balance in the unsaturated and saturated zone is modeled taking into account important phenomena such as hydraulic hysteresis and evapotranspiration.
In this poster the hydrological component of the model will be presented and tested against well established benchmark experiments [Vauclin et al, 1975; Vauclin et al, 1979]. Furthermore, we investigate the applicability of incorporating it in a hydrological catchment model for the prediction (temporal and spatial) of rainfall-triggered shallow landslides.

## Introduction

Cellular Automata are dynamical systems where space, time and states are hypothesized as being discrete. The space is discretized in regular cells and the state of each cell is updated according to a fixed mapping function (local transition function). In complex phenomena, this approach allows us to capture the fundamental characteristics of systems whose global behavior is derived from the collective effect of numerous simple components interacting locally. In addition, temporal and spatial heterogeneity both in the transition function and in the neighborhood of each cell, which characterizes Cellular Automata framework gives us the opportunity to model easily the flow through heterogeneous and anisotropic porous media.

Modelling based in Cellular Automata constitutes a valid alternative to analytical – deductive methods based on the analysis of physical equations describing a particular phenomenon. In order to obtain a discrete formulation of the fundamental equation of a physical theory it is not necessary to go down to the differential form and then go up again to the discrete form. A good approximation is to apply directly the elementary physical laws (in our case the Darcy`s law) to small regions, resulting from the discretization of space, where the uniformity of the field is attained to a sufficient degree [Tonti, 2001; Mendicino et al, 2006].

## Formulation of the model

The model is based on a discrete formulation of the mass balance equation combined with the Darcy`s law (constitutive equation), which can be written in the form:

$$\sum_a -K_{ca}(\psi_c)(\frac{h_a - h_c}{l_a})A_a + V_c C_c \frac{\Delta H_c}{\Delta t} = q_c$$

Where $K_{ac}(\psi_c)$ [$LT^{-1}$] is the hydraulic conductivity averaged between the neighboring cells, $\psi_c$ [L] is cappilary pressure, $h_c$ and $h_a$ [L] are the total heads, $l_a$ [L] is the cell dimension, $A_a$ [$L^2$] is the surface where the flux passes through, $V_c$ [$L^3$] is the cell volume, $C_c(\psi_c)$ [$L^{-1}$] is the specific retention capacity, $\Delta h_c/\Delta t$ is the total head gradient in the time step $\Delta t$ and $q_c$ [$L^3T^{-1}$] is the volumetric mass source term.

In this formulation the hydraulic head is assumed to be continuous and need not to be differentiable. Each cell may have different contitutive properties, which makes it easier to model porous or fractured media. For the unsaturated conditions, it is necessary to specify the nonlinear dependencies between cappilary pressure $\psi_c$ and terms characterizing the hydraulic properties of soil represented by water content $\theta_c$, retention capacity $C_c$ and hydraulic conductivity $K(\psi_c)$.

## Soil hydraulic functions and inter-nodal conductivity

The van Genuchten – Mualem model is one of the most widely used models, despite its possible restrictions. In our model, a modified van Genuchten - Mualem model [Vogel et al, 2001] is used for the description of soil hydraulic functions. The parameter $h_s$ is the minimum capillary height and the parameter $\theta_m$ is fictitious that replaces $\theta_s$ ($\theta_m \geq \theta_s$). This model takes into account the highly non-linear K(h) relationships than can substantially impact the performance of numerical methods in terms of accuracy, stability and rate of convergence. The model is summarized in the following equations:

$$\theta(h) = \begin{cases} \theta_r + \dfrac{\theta_m - \theta_r}{[1+|ah|^n]^m}, & h < h_s \\ \theta_s, & h \geq h_s \end{cases}$$

$$K(h) = \begin{cases} K_s \cdot \left(\dfrac{\theta - \theta_r}{\theta_s - \theta_r}\right)^{0.5} \left[\dfrac{1 - \left(1 - \left(\dfrac{\theta - \theta_r}{\theta_m - \theta_r}\right)^{1/m}\right)^m}{1 - \left(1 - \left(\dfrac{\theta_s - \theta_r}{\theta_m - \theta_r}\right)^{1/m}\right)^m}\right]^2, & h < h_s \\ K_s, & h \geq h_s \end{cases}$$

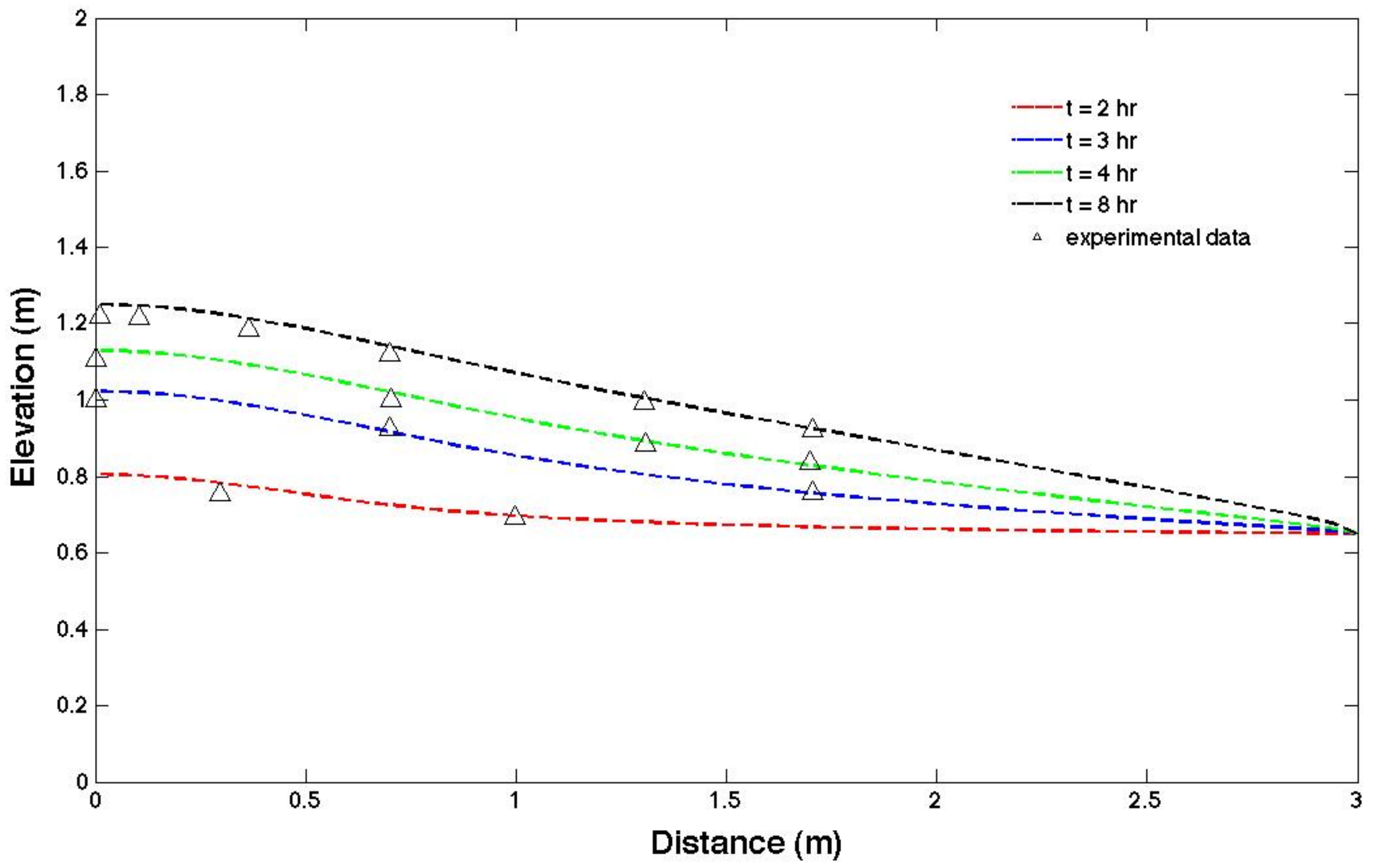
The inter-nodal conductivity is computed as the value of conductivity for arithmetic mean water content: $K_{inter} = K \{ h [ 0.5 ( \theta(h_{i+1}) + \theta(h_i) ) ] \}$. This method seems to produce better results compared to the standard methods of averaging (arithmetic, geometric and harmonic mean).

## Experiment 1: Two-Dimensional Variably Saturated Transient Infiltration Study

The 2-D transient water-table experiment of Vauclin et al (1979) consisted of a *6.0m by 2.0m* box containing a sandy soil with the initial water table located at 0.65m from the bottom. A constant flux of *q = 3.55 m/d* was applied over a width of 1.0 m across the center of the soil surface for 8 hours and the remainder of the surface was covered to prevent evaporative losses.

Due to the experiment`s symmetry, only one-half of the box was modelled. The model domain was 3.0 m by 2.0 m, with the constant-flux boundary condition (*q=3.55 m/d*) applied across the left 0.5 m of the defined ground surface. The initial total head of all cells was set to 0.65 m, and the right boundary cells were constrained to this initial water-table position throughout the 8-hour simulation.

The domain is discretized using $\Delta x = \Delta z = 0.05$ m and the time step is $\Delta t = 1$ sec. The soil used in the experiment was a sandy soil with the following properties: $K_{sat} = 8.4$ m/d, $\theta_s = 0.3$, $\theta_r = 0.033$ and the following van Genuchten-Mualem parameters: $\alpha = 3.3\ m^{-1}$, $n = 4.1$.

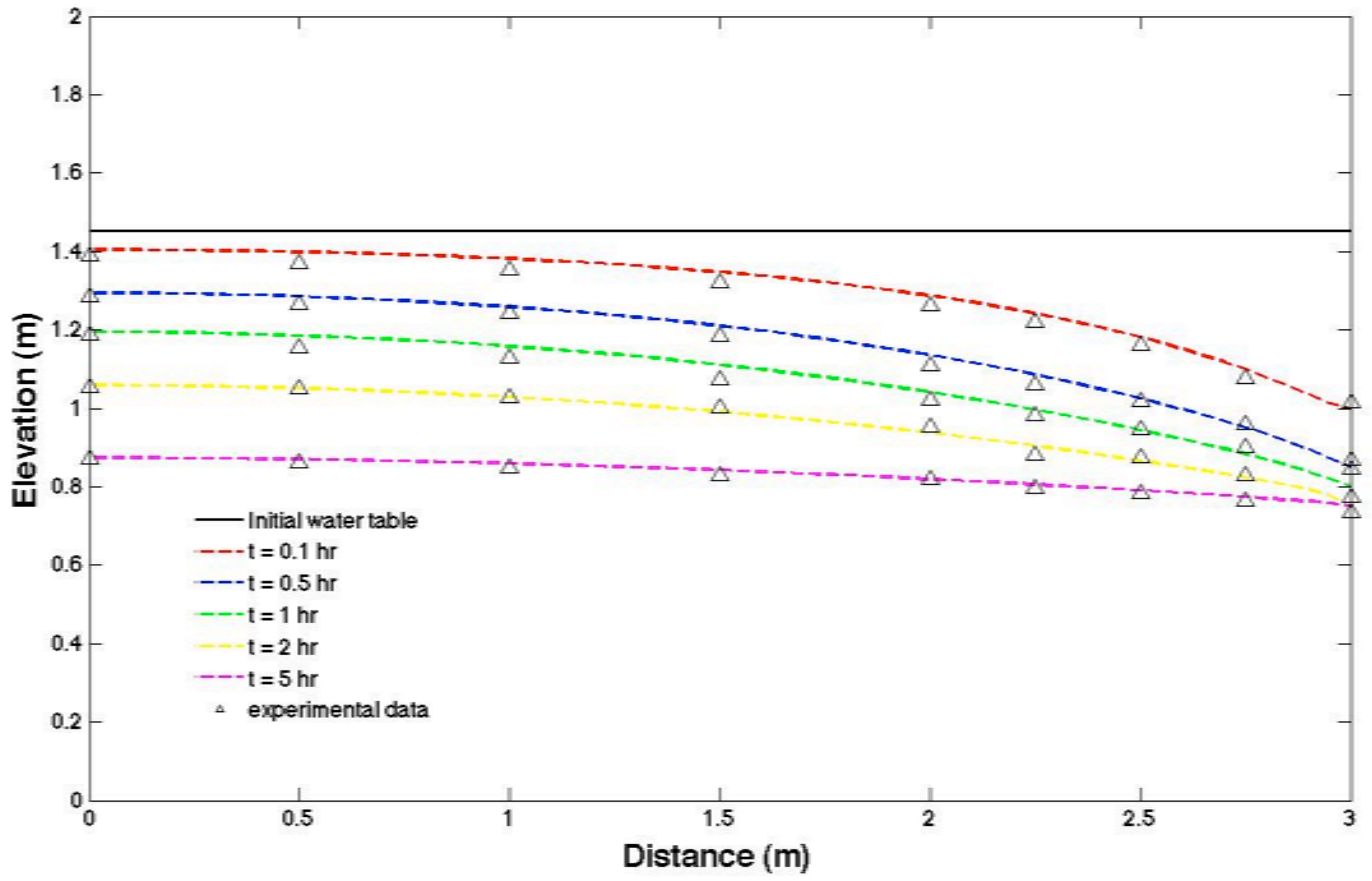


As shown in the figure the model can reproduce the observed transient water table dynamics well. The total CPU time was 105.50 sec. Fahs et al. (2009) presented a numerical scheme based on a Mixed Hybrid Finite Element (MHFE) combined with the Method Of Lines (MOL). The authors report a CPU time of 25.28 sec with a variable time scheme. The efficiency of the CA model could be improved by using a coarser grid, a variable time scheme and by taking advantage of the parallelism that characterises the CA framework. Grid convergence tests are planned for future model testings.

## Experiment 2: Two-Dimensional Variably Saturated Transient Drainage Study

This experiment of Vauclin et al, 1975 examined transient drainage in a fine sandy soil through a seepage face within a rectangular 3.0 m by 2.0 m by 2.0 cm thick soil slab. The soil stab initially was held in hydrostatic equilibrium at h = 1.45 m. The experiment involved measuring the change in moisture content and pressure head as the constant-head boundary imposed on one side of the soil slab instantaneously dropped to h = 0.75 m. The measurements continued until the water table equilibrated to the boundary condition of 0.75 m.

Above the water table the right boundary was set as a seepage face. The first 5 hours of the experiment were simulated. The domain is discretized using $\Delta x = \Delta z = 0.05$ m and the time step is $\Delta t = 1$ sec. The soil used in the experiment was a sandy soil with the following properties: $K_{sat} = 0.4$ m/h, $\theta_s = 0.3$, $\theta_r = 0.0$. The van Genuchten – Mualem parameters were estimated using RETC [van Genuchten et al, 1991]: $\alpha = 2.98\ m^{-1}$, $n = 3.49$.



Also in this case the results show that the model captures satisfactorily the transient water table dynamics. The total CPU time was 75.60 sec.

## Perspectives of future work

This approach can be used for the development of a physically based, distributed model for the spatial and temporal prediction of hydrological triggered shallow landslides. The continuous modelling of water balance is important for a more accurate detection of timing and location of soil slip occurrences. The 3-D flow of water and the resulting water balance in the unsaturated and saturated zone will be modelled using the Cellular Automata framework by taking into account important phenomena such as hydraulic hysteresis, evapotranspiration and the effect of surface runoff. Finally, a geotechnical model will be then coupled to the hydrological component of the model. A first approach could be the adoption of infinite slope analysis which is a quite reasonable assumption considering the conditions prevailing during shallow landslide events.

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